# Uncertainty of Abild's method to determine $U_{50}$.

Søren Ott

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In WAsP Engineering fifty year winds are estimated with a probability weighted moment method suggested (but not invented) by Abild, Mortensen and Landberg (1992). The method uses $n$ years of observations. For each year the maximum is extracted and the $n$ annual maxima $x_1, x_2, \ldots, x_n$ are ordered so that $x_1 < x_2 < \ldots < x_n$. From this set two statistics are formed:

$$Y_1 = \frac{1}{n}\sum x_j \tag{1}$$

$$Y_2 = \frac{2}{n(n-1)}\sum (j-1)x_j \tag{2}$$

$Y_1$ is just the average annual maximum. Assuming that the annual maxima are independent, $Y_2$ is the average bi-annual maximum, obtained by randomly choosing two annual maxima from the sample and discard the smallest one. Assuming a Gumbel distribution

$$F(u) = \exp\left[-\exp\left\{-\frac{u-\beta}{\alpha}\right\}\right] \tag{3}$$

we have

$$\langle Y_1 \rangle = \beta + \alpha\gamma \tag{4}$$

$$\langle Y_2 \rangle = \beta + \alpha\gamma + \alpha \log 2 \tag{5}$$

where $\gamma = 0.5772\ldots$ is Euler's constant. The $T$ year wind is

$$U_T = \beta + \alpha \log T \tag{6}$$

Using $Y_1$ and $Y_2$ as estimates of their respective mean values we can determine $\alpha$ and $\beta$ and find $U_T$ for $T = 50$. This is Abild's method.

Abild gives an estimate of the uncertainty of the method expressed as the variance of the estimated $U_T$:

$$\sigma^2_{U_T} \approx \frac{\alpha^2\pi^2}{6n}(1 + 1.14k_T + 1.1k_T^2) \tag{7}$$

where

$$k_T = \frac{\sqrt{6}}{\pi}(\log\log T/(T-1) - \gamma) \tag{8}$$

This formula is derived using a normal distribution instead of a Gumbel distribution to derive the terms involving $k_T$. In Abild's definition of $k_T$ it would be more correct, and simpler, to replace $\log T/(T-1)$ with $T$.

In order to remove all doubt about the correctness of the error estimate, a Monte Carlo simulation was made to establish an accurate formula. As it is clear the $\sigma_{U_T}$ scales with $\alpha$ and is independent of $\beta$. In fact it can be shown that $\sigma^2_{U_T}/\alpha^2$ is a second order polynomial in $\log T$. We following expression was found to be convenient

$$\sigma^2_{U_T} \approx \frac{\alpha^2\pi^2}{6}\left(\frac{1}{n} + \frac{a_1\, q_T}{n} + \frac{a_2\, q_T^2}{n+n2}\right) \tag{9}$$

where

$$q_T = \frac{\log T - \gamma}{\log 2} \tag{10}$$

Due to the scaling of the variance with $\alpha^2$ and the independence of $\beta$, we may choose to simulate data for a Gumbel distribution with $\alpha = 1$ and $\beta$=0. $10^6$ sets $x_1, \ldots x_n$ were generated for each sample size $n$ ranging from $n = 2$ to $n = 25$. From this the parameters $a_1$, $a_2$ and $n_2$ were then estimated. The result is

$$a_1 = 0.584 \tag{11}$$

$$a_2 = 0.234 \tag{12}$$

$$n_2 = -0.823 \tag{13}$$

# References

Abild, J., Mortensen, N. G. and Landberg, L.: 1992, Application of the wind atlas method to extreme wind data, *J. Wind Eng. Ind. Aerodyn.* **41–44**, 473–484.